Sequential Screening
A first example

- leisure travelers $\nu \sim U[1, 2]$
- business travelers $\nu \sim U[0, 1] \cup U[2, 3]$
- marginal cost of production $c = 1$
- ex post pricing versus ex ante pricing
- price of ticket and refund fee of ticket
Continuous Types

- zero period type $t \in [0, 1]$, first period type $v \in [0, 1]$
  \[
  \max_{x(t,v)y(t,v)} \int \int f(t) x(t, v) - cy(t, v) g(v \mid t) \, dvdt
  \]
- subject to sequential incentive constraints $t = 1$:
  \[
  \forall t, \forall v, v'; \quad vy(t, v) - x(t, v) \geq vy(t, v') - x(t, v')
  \]
- and $t = 0$: $\forall t, t'$
  \[
  \int f(t) x(t, v) - cy(t, v) g(v \mid t) \, dvdt \\
  \geq \\
  \int f(t) x(t', v) - cy(t', v) g(v \mid t) \, dvdt
  \]
- participation constraint (interim for all $t$)
  \[
  \int (vy(t, v) - x(t, v)) g(v \mid t) \, dv
  \]
Lemma

The second period incentive compatibility constraints are satisfied if and only if

1. \( \frac{\partial u(t, v)}{\partial v} = y(t, v) \)
2. \( y(t, v) \) is nondecreasing in \( v \) for each \( t \).

but now we have only necessary conditions for the first period

Lemma

The first period incentive compatibility constraints are satisfied only if:

1. the information rent evolves
   \[
   \frac{dU(t)}{dt} = - \int y(t, v) \left( \frac{\partial G(v | t)}{\partial t} \right) dv,
   \]
2. the cumulative delivery evolves:
Multidimensional Mechanism Design

Theorem

Suppose $T$ is ordered by FSD. If a delivery rule $y(t,v)$ solves the relaxed problem, and if $y(t,v)$ is non-decreasing in $t$ for all $v$ and in $v$ for all $t$, then there exist transfer payments $x(t,v)$ such that the sequential mechanism \{$y(t,v), x(t,v)$\} is optimal.

- we next discuss an example as to how these condition is violated...
Counterexample (for Missing Sufficient Conditions)

- $t$ is drawn from $F$
- $v = t$ with probability $p$; with probability $1-p$ it is drawn from $F$ independently
- the impulse response is 1 if $v = t$, otherwise it is 0.
- when solving the relaxed problem (i.e., maximizing dynamic virtual) we get an allocation that is first-best for any $(v, t)$ such that $v \neq t$, but there is a distortion if $v = t$
- this allocation is not implementable, which is the easiest to see by noting that for any fixed $t$, the allocation should be monotone in $v$, but now we have something non-monotone.
Recall the Pricing Conditions of the Relaxed Constraints

• the conditions for \( t = 0 \) and \( t = 1 \):

\[
p_0 (\theta_0) = \theta_0 + (1 - \alpha) \frac{F_0(\theta_0)}{f_0(\theta_0)}, \]
\[
p_1 (\theta_0, \theta_1) = \theta_1 - (1 - \alpha) \frac{F_0(\theta_0)}{f_0(\theta_0)} \frac{\partial F_1(\theta_1|\theta_0)}{\partial \theta_0}.
\]

• in particular if \( \theta_0 = \theta_1 \), then

\[
p_1 (\theta_0, \theta_1) = \theta_1 - (1 - \alpha) \frac{F_0 (\theta_0)}{f_0 (\theta_0)} p
\]

and if \( \theta_0 \neq \theta_1 \), then

\[
p_1 (\theta_0, \theta_1) = \theta_1
\]

• but now the perior \( t = 1 \), cannot be monotone
Restatement of the Sufficient Condition

- consider the Lemma 3.3 of Courty and Li

**Lemma**

*Suppose that $T$ is ordered by FSD. If a delivery rule $y(t, v)$ solves the relaxed problem, and if $y(t, v)$ is non-decreasing in $t$ for all $v$ and in $v$ for all $t$, then there exist transfer payments $x(t, v)$ such that the sequential mechanism $\{x(t, v), y(t, v)\}$ is optimal.*

- issue: even with FSD, the allocation $y(t, v)$ is not guaranteed to be monotone increasing in along each dimension, and hence the statement is still not in terms of primitives (but in terms of optimality conditions).