1. **Optimal Static Regulation.** Consider the static model of (Baron and Myerson 1982) of optimal regulation, where the social surplus is given by

\[ V(q) - pq + \alpha (pq - \theta q), \]

and the marginal cost parameter is assumed to be distributed with a continuously differentiable distribution function \( F(\theta) \) on a compact set \( \Theta \subset \mathbb{R}_+ \).

(a) Describe the indirect utility function of the regulated firm in any incentive compatible direct mechanism, where the ex post participation constraint of the firm requires that the firm receives a nonnegative profit for every type \( \theta \):

\[ \pi(\theta, p(\theta), q(\theta)) = (p(\theta) q(\theta) - \theta q(\theta)) \geq 0. \]

(b) Present sufficient conditions on \( F(\cdot) \) so that a social surplus maximizing allocation is monotone decreasing in \( \theta \), the marginal cost parameter.

(c) Explicitly derive the social surplus maximizing allocation \((q^*(\theta), p^*(\theta))\) by means of pointwise maximization.

2. **Optimal Static Regulation with Ex Ante Participation Constraint.** Suppose now that the ex post participation constraint is weakened to the ex ante participation constraint, which only requires that:

\[ \mathbb{E}_0 [\pi(\theta, p(\theta), q(\theta))] = \mathbb{E}_0 [p(\theta) q(\theta) - \theta q(\theta)] \geq 0. \]

What is the social surplus maximizing mechanism now? Does it depend on \( \theta \)? (See (Baron 1981) for additional insights.)

3. **Optimal Dynamic Regulation.** In the discussion of (Baron and Besanko 1984), we described in class how the informativeness of the private signal in period 0 enters period 1. Extend the informativeness to a regulation problem with finitely many period \( T > 2 \). Present an argument as to why intermediate expression between \( t \) and \( s \) with \( 0 < t < s \leq T \) do not appear in the informativeness, or the first order conditions.
4. **Optimal Dynamic Regulation.** Consider the following explicit model of two period regulation, where $\theta_0 \sim \mathcal{U}[0, 1]$ and $\theta_1 \sim \mathcal{U}[0, 1]$ with $\varepsilon \in (0, 1)$

$$\theta_1 \sim \begin{cases} 
2 - \theta_0, & \text{if } \theta_1 \in [0, \frac{1}{2}] \\
\theta_0, & \text{if } \theta_1 \in [\frac{1}{2}, 1].
\end{cases}$$

(a) Does the distribution $F_1 (\cdot)$ satisfy First Order Stochastic Dominance for all $\varepsilon$?

(b) Does the distribution $F_1 (\cdot)$ satisfy the monotonicity condition.

(c) Derive the optimal dynamic regulatory scheme using only the local incentive constraints. Can you verify that the global incentive constraints are also satisfied.

**References**

