Problem Set 1

1. Consider a game where $N$ potential employers compete for the services of a single worker. The worker works in one of the firms in each period. At the outset, her productivity is unknown. Firms have private prior information on this. The worker’s productivity in firm $i$ is $v_i$ with probability $p_i$ and the productivity is zero with complementary probability $(1 - p_i)$. Assume that the productivities are independent across firms. The worker’s productivity in firm $i$ is revealed after working a period in firm $i$. All players discount future with a constant discount factor $\delta < 1$.

(a) Solve the efficient allocation policy of the worker to the firms under the assumption of complete information.

(b) Compute the firms’ marginal contribution to social surplus and derive the transfers for the Dynamic Pivot Mechanism (DPM) for the incomplete information case.

(c) (To be done after Dirk has talked about revenue maximization). Suppose the mechanism designer wants to maximize revenue. How can you improve on the DPM?

2. Consider a problem of scheduling where a single unit of capacity is allocated to serve tasks for two potential bidders $i = 1, 2$. The value of service is $\omega_1$ for the first bidder’s task and bidder 1 has an infinite sequence of such tasks. The value of service for player 2 is $\omega_2$ and bidder 2 has only a single such task. Suppose the $\omega_i$ are private information to the bidders and that the values are independent across players (but all tasks of bidder 1 have the same value $\omega_1$).
(a) Compute the DPM for this case.

(b) Suppose the capacity is auctioned in each period according to an ascending price auction. Construct a Perfect Bayesian Equilibrium for the overall *infinite horizon) game. Is the equilibrium of this game efficient?

3. Consider a scheduling problem where each of \( N \) bidders has a single-period task worth \( \omega_i \). Show that the marginal contributions of the bidders are superadditive in the sense that for all subsets of bidders \( S, T \subset \mathcal{N} \) with \( S \cap T = \emptyset \), we have

\[ M_S + M_T \leq M_{S \cup T}. \]

4. (Static Mechanism Design) Assume that we are in a private values setting and that the conditions for Revenue Equivalence Theorem hold and as a result, the VCG -mechanisms are the only efficient dominant strategy incentive compatible mechanisms. Show that there exists an efficient balanced budget mechanism (where incentive compatibility is understood in the dominant strategy sense) if and only if for every agent \( i \in \{1, \ldots, N\} \), there exists a function \( f_i : \Theta_{-i} \rightarrow \mathbb{R} \) such that:

\[ \sum_{i=1}^{N} u_i (q(\theta), \theta_i) = \sum_{i=1}^{N} f_i (\theta_{-i}) \text{ for all } \theta \in \Theta. \]