Targeting in Advertising Markets: Implications for Offline vs. Online Media

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“Recent” progress in advertising technology:
- display advertising
- sponsored search
- social networks
- mobile ads
- addressable cable
... 

Distinctive feature is the ability to target:
- attribute, demographic targeting;
- behavioral, contextual targeting.
to offer a model of targeting in advertising markets in the presence of

- many distinct advertising markets
- many distinct advertisers

we trace out the implications of targeting for:

- the allocation of advertisement messages;
- the social value of advertising;
- the equilibrium price of advertising;
- the equilibrium revenues of new and old media.
A Model of Advertising as Matching

- Advertising matches a consumer and a product.
- An advertisement message turns a potential, interested consumer into an actual customer.
- Advertising markets operate under substantial frictions:
  1. messages may reach the wrong consumer;
  2. messages may reach the same consumer repeatedly.
- Targeting reduces matching frictions.
A continuum of distinct advertising markets

\[ a \in [0, \infty), \]

representing outlets, channels, websites, searches.

A continuum of distinct products (\(=\) firms),

\[ x \in [0, \infty). \]

A unit mass of consumers with two-dimensional type \((a, x)\):

- each consumer is located in a specific advertising market \(a\);
- each consumer is interested in a specific product \(x\).
A consumer is characterized by \((a, x)\):

1. his location in a specific advertising market \(a\),
2. his preference for a specific product \(x\)

market structure: joint density \(s(a, x)\) over \((a, x)\):

\[
\int_{0}^{\infty} \int_{0}^{\infty} s(a, x) \, da \, dx = 1.
\]
advertising market \( a \): distribution over consumer preferences

\[
s (x | a) = \frac{s(a, x)}{\int_0^\infty s(a, x') \, dx'}
\]

firm \( x \): distribution of its consumers over advertising media:

\[
s (a | x) = \frac{s(a, x)}{\int_0^\infty s(a', x) \, da'}
\]
we maintain the distribution over consumer preferences:

$$s(x) = \int_{0}^{\infty} s(a', x) \, da',$$

the share $s(x)$ of each product in the consumer market

we order $x$ (without loss of generality) so that:

$$s'(x) < 0,$$

there are products with a broad audience $x \approx 0$ and products with a narrow audience $x \approx \infty$ (the long tail of Anderson (2006))
we investigate the impact of different distributions of consumers across advertising markets.

the distribution of consumer across advertising markets range from perfect targeting to zero targeting.

and ask how does an increase in targeting impact the allocation and the price of advertising across media markets.
an increase in targeting then has two effects:

1. consumers move from mass market publications to more specialized, narrower media
2. in every media market, the naturally targeted audience has a larger relative population share
Exponential distribution of consumers’ interests:

\[ s_x := \lambda e^{-\lambda x}. \]

\( \lambda \) measures concentration of consumers in product markets.

Market shares \( s_x \) are declining in \( x \).

Hierarchical structure of products:

- popularity: bicycles, music, watches, travel destinations;
- mass vs. niche products, mainstream vs. fringe firms.
Conditional distribution of consumers $x$ in markets $a$:

$$s(x | a) = \gamma e^{-\gamma(x-a)}, \quad \text{for all } 0 < a \leq x.$$ 

Distribution across markets is upper triangular (stationary):

$$s(x | a) = 0 \quad \text{for all } x < a,$$
an increase in the targeting technology $\gamma$ has a size effect...

- $\gamma$ measures consumer concentration in advertising markets.
- High $\gamma \Rightarrow$ the consumers of $x$ move to nearby markets $a \approx x$. 
Composition of Advertising Markets

... and an increase in targeting $\gamma$ has a composition effect:

- $\gamma$ measures consumer concentration in advertising markets.
- high $\gamma \Rightarrow$ most consumers in $a$ have nearby preferences $x \approx a$.
- a higher $\gamma$ facilitates targeting.

Parameter Values: $\lambda=1/4$, $\gamma \in \{1,3\}$
Advertising as Random Matching

- Each consumer reads/views/processes $M$ messages
- A consumer with preference for product $x$ purchases if and only if she receives a message from firm $x$
- Firm $x$ sends $m_{a,x}$ messages to consumers in market $a$.
- Each message is received with uniform probability by one of the consumers in advertising market $a$.
- It follows that a consumer in advertising market $a$ receives at least one message from firm $x$ with probability

$$f(m_{a,x}, s_a) = 1 - \exp\left(-\frac{m_{a,x}}{s_a}\right).$$
an advertising policy of firm $x$:

$$\{ m_{a,x} \}_{a=0}^x$$

advertising intensity in advertising market $a$:

$$\frac{m_{a,x}}{s_a}$$

determine the gross revenue of $m_{a,x}$ is given by

$$s_{a,x} \cdot f (m_{a,x}, s_a) = s_{a,x} \cdot (1 - \exp (-m_{a,x} / s_a))$$

an optimal advertising policy seeks to minimize the role of:

1. irrelevant messages: $1 - s_{a,x}$
2. duplicating messages: $\exp (-m_{a,x} / s_a)$
price of message in advertising market $a$ is competitive equilibrium price $p_a$

$M$ is time/attention of consumer devoted to advertisements

supply of messages $M_a$ in advertising market $a$ is given by:

$$M_a = s_a \cdot M$$

competitive price $p_a$ equilibrates demand and supply:

$$\int_0^\infty m_{a,x}(p_a) \, dx = M_a.$$
The Firms’s Problem

- Each sale generates revenue $1, firms only differ in size $s(x)$.

- Firm $x$ chooses $m_{a,x}$ to maximize profit:

$$\pi_{a,x} = \max_{m_{a,x}} \left[ s_{a,x} \cdot \left( 1 - \exp \left( - \frac{m_{a,x}}{s_a} \right) \right) - p_a \cdot m_{a,x} \right].$$

- Advertising policies are separable across advertising markets:

$$m_{a,x} = s_a \left( \ln \frac{\gamma + \lambda}{p_a} - (\gamma + \lambda) (x - a) \right)$$

for all $x \geq a$. 
Marginal advertiser in advertising market \( a \) is \( X_a^* \).

The number of active firms is constant across markets \( a \):

\[
X_a^* - a = \sqrt{\frac{2M}{\lambda + \gamma}}.
\]

The equilibrium demands are

\[
m_{a,x}^* = \gamma \lambda e^{-a\lambda} (X_a^* - x).
\]
• Equilibrium prices $p^*_a$ are equalized across advertising market:

$$p^*_a = p^* = (\lambda + \gamma) e^{-\sqrt{2(\lambda + \gamma)M}}, \text{ for all } a.$$ 

• For any $\gamma > 0$, all firms advertise somewhere positive targeting $\Rightarrow$ “long tail”.
The Social Value of Targeting

- an improvement in targeting technology as increase in $\gamma$
- what is the impact in terms of the social welfare?
  - less irrelevant messages are received
  - more messages are sent by smaller firms

Proposition (Targeting and Social Welfare)

As targeting improves the social value of advertising increases.

- the total number of matches between advertisers and consumers increases
- even, the number of matches of each firm (product) increases
as the social value of advertising increases, how does the composition in the demand for advertising change?

Proposition (Targeting and Demand)

As targeting improves:

1. the large firms purchase less, the small firm purchase more messages (across all markets);

2. the number of participating firms $X^*_a$ decreases in every advertising market;

3. The number of messages per capita $m^*_{a,x} / s_a$ increases for all $x < (a + X^*_a) / 2$.

conversely, every firm is present in fewer advertising markets
as the social value of advertising increases, can (a share of) the increase in value be captured by the media?

Proposition (Targeting and Price)

As $\gamma$ increases, the equilibrium price per message $p^*_a$ increases if and only if $\lambda + \gamma < 2/M$.

- the equilibrium price is initially increasing in the targeting ability but eventually decreasing
- main trade-off: the messages become more relevant yet eventually to a smaller set of firms and thus the risk of duplication (saturation)
- in hedonic terms: the price per consumer reached is decreasing everywhere.
- advertiser value more homogenous groups of readers (in subscriber characteristics of age, gender, income, etc.)

Rutz and Bucklin (2010): "From Generic to Branded: A Model of Spillover Dynamics in Paid Search Advertising,” compare generic (e.g., “Hotels LA”) and branded (e.g., “Hilton Hotels LA”) searches
- find that branded keywords have lower prices than generic keywords "Sheraton Hotel NYC" vs "Hotel NYC;
- find that long, narrower keywords “Hotels LA Westwood” have lower prices than shorter ones “Hotels LA"
allow for multi-homing of consumer and thus multiple opportunities for advertiser to match with a customer

online versus offline media, targeted vs. non-targeted medium

total exposure to advertising, given by $M$, is now divided between media, $A$ and $B$:

$$M_A + M_B = M$$

suppose firm $x$ reaches a fraction $a_x$ of its consumers on medium $A$, and a fraction $b_x$ on medium $B$.

the total fraction of $s_x$ reached is

$$a_x + b_x - a_x \cdot b_x.$$
Online vs. Offline Media

- general (offline, $A$) and perfectly targeted (online, $B$) advertising $(m^A_x, m^B_{a,x})$.

- supply in the (single) offline market is $M_A$.

- supply in online market $a$ is $M_{B,a} := s_a \cdot M_B$.

- perfectly targeted advertising online: $\gamma = \infty$

- the relevant online advertising market for firm $x$ is $a = x$. 
large firms \((x < X^*)\) are present online and offline
small firms \((x > X^*)\) are present only online

**Proposition (Equilibrium Prices)**

1. **The equilibrium price on the offline medium is given by:**
   \[ p^* = \lambda \exp(-M_B - \sqrt{2\lambda M_A}). \]

2. **The equilibrium prices on the online markets are given by:**
   \[ p_a^* = \begin{cases} 
   \exp(\lambda a - M_B - \sqrt{2\lambda M_A}), & \text{for } a \leq X^*, \\
   \exp(-M_B), & \text{for } a > X^*. 
   \end{cases} \]
The Emergence of the Internet

- the attention/time allocated to online media, $M_B$, is increasing; conversely the attention to offline media, $M_A$ is decreasing
- the segment of firms advertising offline is shrinking as $x < X^*$:
  \[ X^* = \sqrt{\frac{2M_A}{\lambda}} \]
- the price of advertising offline is decreasing faster, linear rather than square root, with an increase in the online media:
  \[ p^* = \lambda \exp(-M_B - \sqrt{2\lambda M_A}) \]
- in particular, relative to the introduction of competing offline medium where it would be:
  \[ p^* = \lambda \exp(-\sqrt{2\lambda (M_A + M_B)}) \]

when lawyers cannot contact clients by mail, advertising prices per click for search engine advertisements are 5-7% higher. Therefore, online advertising substitutes for offline advertising consistent with Chandra and Kaiser (2010) who document the positive valuation of homogenous, targeted audiences; and hence imply differential revenue across media with differential targeting abilities.
A model of targeting in competitive advertising markets.

Hierarchical framework for product and advertising markets.

Extensions and future directions:

1. revenue maximization, strategic interaction;
2. platform competition;
3. congestion, consumer preferences over for different ads;
4. ad exchanges.