Information Acquisition in Interdependent Value Auctions

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role of private information in mechanism design
agents have private information that is relevant for (efficient) allocation
designer defines mechanism to elicit private information
information revelation is voluntary (incentive compatibility)
Information Acquisition

- key assumption in mechanism design literature:
  - private information is exogenously given

- our paper allows information to be privately acquired:
  - social value of information
  - equilibrium value of information

- examples: oil tracts & license auctions
  - private information acquired through costly investment
  - interdependent values
each agent privately decides to acquire information:
  - ex-ante
  - covertly

information structure is endogenous
  - ex-post mechanism affects incentives to acquire information ex-ante
  - spectrum licenses: lottery vs. auction

is it possible to design mechanisms that perform well:
  - ex-ante
  - ex-post
Current Paper

- information acquisition in ex-post efficient mechanisms
- generalized Vickrey-Clarke-Groves (VCG) mechanism
- whether and how equilibrium information acquisition differs from the social optimum
- how the difference depends on:
  - the strength of the interdependence
  - the number of informed bidders
private values setting
Stegeman (1996) considers second price auctions
Bergemann and Välimäki (2002) consider general allocation problems
each agent receives in equilibrium his marginal contribution
each agent has correct incentives to acquire information
information aggregation and costly information acquisition

- Milgrom (1981): Vickrey auction
- Jackson (2003): informational efficiency is not robust to cost of information

interdependent values setting:

- Maskin (1992) considers second price auction
- Bergemann and Välimäki (2002) consider general allocation problems
  - given decisions of other agents (locally), individual incentives are socially excessive (insufficient) if valuations are positively (negatively) dependent
Main Results

- provide a comparison of *equilibrium* level and social optimal level of information
  - information decisions are strategic substitutes
  - positive dependence: equilibrium information is socially excessive

- difference between socially optimal and equilibrium level decreases if
  - more agents acquire information
  - level of positive dependence decreases
Model

- auction setting with interdependent values
- single object and \( I \) bidders
- value to bidder \( i \) is linear in bidders’ signals \( \{\theta_i\}_{i=1}^{I} \):

\[
 u_i (\theta_i, \theta_{-i}) = \theta_i + \alpha \sum_{j \neq i} \theta_j,
\]

where \( 0 \leq \alpha \leq 1 \) measures interdependence
- quasilinear utility:

\[
 u_i (\theta) - t_i,
\]

where \( t_i \) is monetary transfer
Information

- \( \theta_i \)'s are i.i.d. from a common prior \( F \) with support \([\theta, \bar{\theta}]\) and

\[
\mu = \mathbb{E}[\theta_i]
\]

- private information \( \theta_i \) unknown ex ante
- binary information decision:
  - if bidder \( i \) acquires information, \( i \) privately observes \( \theta_i \)
  - otherwise, \( i \)'s information is given by prior \( F \)

- information cost \( c > 0 \)
two-stage game:
  - information acquisition stage
  - bidding stage

direct revelation mechanism $\{q_i, t_i\}_{i=1}^L$

generalized Vickrey-Clarke-Groves mechanism:

$$y_i = \max_{j \neq i} \{ \theta_j \}$$

then the allocation rule is

$$q_i (\theta_i, \theta_{-i}) = \begin{cases} 
1 & \text{if } \theta_i > y_i \\
0 & \text{if } \theta_i < y_i
\end{cases}$$

and the payment rule

$$t_i (\theta_i, \theta_{-i}) = \begin{cases} 
0 & \text{if } \theta_i < y_i \\
u_i (y_i, \theta_{-i}) & \text{if } \theta_i > y_i
\end{cases}.$$
two bidders: $i$ and $j$

$$u_i (\theta_i, \theta_j) = \theta_i + \alpha \theta_j$$

with $\alpha \in (0, 1)$.

in the generalized VCG mechanism the allocation is

$$q_i (\theta_i, \theta_{-i}) = 1 \{ \theta_i \geq \theta_j \}$$

and the transfer is

$$t_i (\theta_i, \theta_{-i}) = (\theta_j + \alpha \theta_j) \cdot 1 \{ \theta_i \geq \theta_j \}$$
Social and Private Payoffs

VCG Allocation and Transfer

\[ u_i(\theta_i, \theta_j) = \theta_i + 0.5\theta_j, \quad t_i(\theta_i, \theta_j) = 1.5\theta_j, \quad \theta_j = 0.5 \]

Social and Private Payoff

\[ \text{max}\{u_i(\theta_i, \theta_j), u_j(\theta_i, \theta_j)\} \]
Social and Private Incentives

Social Gain from Information: A
Private Gain from Information: A+B

\[ u_i(\theta_i, \theta_j), u_j(\theta_i, \theta_j) \]

\[ u_i(\theta_i, \theta_j) = \theta_i + 0.5\theta_j, \quad t_i(\theta_i, \theta_j) = 1.5\theta_j, \quad \theta_j = 0.5, \quad \theta_i = 0.9 \]
set of informed agents: \( \{1, 2, \ldots, m\} \)

set of uninformed agents: \( \{m + 1, \ldots, l\} \)

marginally informed agent: \( m \)

bidder \( h \) has highest signal among agents \( 1, 2, \ldots, m - 1 \):

\[
\theta_h \triangleq \max \{\theta_1, \ldots, \theta_{m-1}\}.
\]
Socially Efficient Information Policy

- $\Delta^*_m$ is expected social gain of marginal informed bidder $m$:

$$
\Delta^*_m = \mathbb{E}_\theta [(u_m(\theta) - u_h(\theta)) \cdot 1(\theta_m \geq \theta_h \geq \mu)] \\
+ \mathbb{E}_\theta [(u_m(\theta) - u_I(\theta)) \cdot 1(\theta_m \geq \mu > \theta_h)]
$$

- $\Delta^*_m$ is the difference between:
  - social value when allocation incorporates information $\theta_m$
  - social value without incorporating information $\theta_m$

- define

$$
y_m = \max \{\theta_h, \mu\} = \max_{j \neq i} \theta_j,
$$

then we have using linearity

$$
\Delta^*_m = (1 - \alpha) \mathbb{E}_{\theta_m, y_m} [(\theta_m - y_m) \cdot 1(\theta_m \geq y_m)]
$$
social efficient policy $s^*_m \in \{0, 1\}$:
- $s^*_m = 1$ if it is efficient to acquire information
- $s^*_m = 0$ otherwise

**Proposition**

The socially efficient policy $s^*_m$ is given by

$$s^*_m = \begin{cases} 
0 & \text{if } \Delta^*_m < c \\
1 & \text{if } \Delta^*_m \geq c
\end{cases}.$$ 

$\Delta^*_m$ is strictly decreasing in $m$ and $\alpha$. 

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Information Acquisition in Interdependent Value Auctions
Equilibrium Value of Information

\( \hat{\Delta}_m \): expected private gain of bidder \( m \) from information about \( \theta_m \)

\[
\hat{\Delta}_m = \mathbb{E}_{\theta} [(u_m (\theta_m, \theta_{-m}) - u_h (y_m, \theta_{-m})) \cdot 1(\theta_m \geq y_m)] \\
= \mathbb{E}_{\theta_m, y_m} [(\theta_m - y_m) \cdot 1(\theta_m \geq y_m)]
\]

\( \hat{\Delta}_m \) is the difference between:

- private payoff of allocation that incorporates information \( \theta_m \)
- private payoff of allocation without incorporating \( \theta_m \)
Proposition

The equilibrium policy in the pure strategy equilibrium is given by

\[ \hat{s}_m = \begin{cases} 
0 & \text{if } \hat{\Delta}_m < c \\
1 & \text{if } \hat{\Delta}_m \geq c 
\end{cases} . \]

\( \hat{\Delta}_m \) is strictly decreasing in \( m \) and constant in \( \alpha \) for all \( m \).
Theorem

For all $m$,

1. private gains are higher than social gains of information;
2. information decisions are strategic substitutes;
3. unique pure strategy equilibrium displays socially excessive information acquisition;
4. the difference $\Delta_m - \Delta^*_m$ is increasing in $\alpha$.

- with positive dependence, equilibrium information is socially excessive
  - the number of informed bidders in equilibrium is larger than in a planner’s solution
- information decisions are strategic substitutes in both equilibrium and social optimum
Mixed Strategy Equilibrium

- symmetric equilibrium
- restrict social program to choose the same probability of acquiring information for all bidders
  - concentrate solely on the informational externalities
  - ignore coordination problems arising due to mixing
- comparison between social and equilibrium level of information continues to hold with symmetric solutions
  - $\sigma^*$: socially optimal probability of acquiring information
  - $\hat{\sigma}$: equilibrium probability of acquiring information
Mixed Strategy Equilibrium

- $\Delta^* (\sigma)$: expected social gain of additional informed bidder
  \[
  \Delta^* (\sigma) = \sum_{m=1}^{I-1} \binom{I-1}{m-1} \sigma^{m-1} (1 - \sigma)^{I-m} \Delta_m^*
  \]

- $\hat{\Delta} (\sigma)$: individual gain if other bidders acquire information with probability $\sigma$

**Proposition**

*For all $\sigma^* \in (0, 1)$, $\sigma^* < \hat{\sigma}$.*
question:

- can we generalize results in the linear setting to a nonlinear environment?

no-crossing condition

- the ranking of any two bidders is unaffected by the private information of a third bidder

example: linear signal model with constant absolute risk aversion utility
Basic Setup

- general nonlinear valuation functions
  \[ u_i : [\theta, \theta'] \rightarrow \mathbb{R} \]

- symmetric: \( \forall \theta, \theta', \) if \( \theta' \) is a permutation of \( \theta \) and \( \theta_i = \theta'_j \), then
  \[ u_i(\theta) = u_j(\theta') \]

- single-crossing property
  \[ \theta_i \geq \theta_j \Rightarrow u_i(\theta) \geq u_j(\theta) \]

- positive interdependence
  \[ \frac{\partial u_i(\theta)}{\partial \theta_j} > 0, \ \forall i, j, \forall \theta. \]
valuations \( \{u_i(\theta)\}_{i=1}^{l} \) satisfy the no-crossing condition if for all \( m \) and all \( i, j \neq m \):

\[
\exists \theta_m \text{ s.th. } \mathbb{E} [u_i(\theta) | \theta_1, \ldots, \theta_m] > \mathbb{E} [u_j(\theta) | \theta_1, \ldots, \theta_m] \quad \Rightarrow \\
\forall \theta_m \text{ s.th. } \mathbb{E} [u_i(\theta) | \theta_1, \ldots, \theta_m] > \mathbb{E} [u_j(\theta) | \theta_1, \ldots, \theta_m]
\]

this condition is important to ensure \( \Delta_m^* < \hat{\Delta}_m \):

- if violated, the information of agent \( m \) may be socially valuable in determining allocation between \( i \) and \( j \) without agent \( m \) ever getting the object
- agent \( m \) will have very weak incentive to acquire information even though it would be socially valuable
- social gain from information about \( \theta_m \) may exceed private gain
Excessive Private Incentives

- no-crossing: curves $E[u_i(\theta) | \theta_1, \ldots, \theta_m]$ and $E[u_j(\theta) | \theta_1, \ldots, \theta_m]$ do not cross
- single-crossing: curve $E[u_m(\theta) | \theta_1, \ldots, \theta_m]$ crosses both $E[u_i(\theta) | \theta_1, \ldots, \theta_m]$ and $E[u_j(\theta) | \theta_1, \ldots, \theta_m]$ only once
- difference between private and social incentives: shaded area
Results

Theorem

If the no-crossing condition is satisfied then

1. the private gain from information is higher than social gain from information \( (\hat{\Delta}_m \geq \Delta^*_m) \);

2. information decisions are strategic substitutes \( (\hat{\Delta}_{m-1} \geq \hat{\Delta}_m) \);

3. unique pure strategy equilibrium displays socially excessive information acquisition.
we identified sufficient conditions for excessive equilibrium information

- private incentives > social incentives
- strategic substitutes

question

- positive interdependence ⇒ excessive equilibrium information?
- not true in general
value of object is determined by the $K$ highest signals.

\[ u_i (\theta) = \theta_i + \alpha \sum_{k=1}^{K} y_{ik} \]

example: license to operate in $K$ markets

- bidder $i$'s signal reveals the profitability of market $i$
- choose to operate in the $K$ markets with highest potential
Privately versus Socially Pivotal Signals

- privately vs. socially pivotal signals
  - privately pivotal: determine the winner of the license
  - socially pivotal: determine which market to operate
  - a signal could be socially pivotal but not privately pivotal

- findings:
  - information decision remain strategic substitutes
  - equilibrium level of information is socially insufficient.
Strategic Complements

- Local comparison may not extend to equilibrium comparison.
- Strategic complements $\Rightarrow$ multiple equilibria.
- Despite positive interdependence, an equilibrium of the game may display a lower level of information acquisition than the social optimum.
two bidders, \( i \in \{1, 2\} \), compete for an object

linear payoff structure: \( u_i(\theta_i, \theta_j) = \theta_i + \frac{1}{2} \theta_j \)

types \( \theta_i, \theta_j \) are independently drawn from \( U [-5, 1] \)

efficient allocation: assign the object to bidder \( i \) if

\[
\mathbb{E} [u_i(\theta)] > \max \left\{ 0, \mathbb{E} [u_j(\theta)] \right\}
\]

otherwise retain the object

information decisions are strategic *complements*

for small \( c \) the efficient policy asks both bidders to acquire information, but in one of the two pure strategy equilibria, both bidders remain uninformed
Conclusion

- with interdependent values equilibrium information differs from social optimum.

- extensions:
  - multi-unit auction setting
  - negative interdependence: too low incentives

- future research questions:
  - how should a planner correct the incentives? participation fees, randomization?
  - revenue maximizing design
  - sequential information design
  - information acquisition in double auctions with large number of traders