

# Large Market Asymptotics for Differentiated Product Demand Estimators with Economic Models of Supply

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August 3, 2015

## Abstract

IO economists often estimate demand for differentiated products using data sets with a small number of large markets. This paper addresses the question of consistency and asymptotic distributions of IV estimates as the number of products increases in some commonly used models of demand under conditions on economic primitives. I show that, in a Bertrand-Nash equilibrium, product characteristics lose their identifying power as price instruments in the limit in certain cases, leading to inconsistent estimates. The reason is that product characteristic instruments achieve identification through correlation with markups, and, depending on the model of demand, the supply side can constrain markups to converge to a constant quickly relative to sampling error. I find that product characteristic instruments can yield consistent estimates in many of the cases I consider, but care must be taken in modeling demand and choosing instruments. A Monte Carlo study confirms that the asymptotic results are relevant in market sizes of practical importance.

## 1 Introduction

The simultaneous determination of quantity and prices is a classic problem in demand estimation. A common solution in markets with differentiated products is to use characteristics

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of competing products as a source of exogenous variation in prices. The idea is that a firm facing stiffer competition will set lower markups, so, as long as they are independent of demand shocks, characteristics of competing products will be valid instruments for a product's price. The use of characteristics of competing products as price instruments is common in empirical studies of differentiated product markets, and goes back at least to Bresnahan (1987) and Berry, Levinsohn, and Pakes (1995) (hereafter, BLP). I refer to instruments of this form as product characteristic instruments or, in reference to the latter paper, "BLP instruments." See footnote 1 below for more recent references that use BLP instruments.

Many empirical studies of markets with differentiated products, including those that use the BLP instruments, use data on a relatively small number of markets, each with many products, to estimate demand elasticities. For example, in their application to automobile demand, BLP use data on 20 markets, each with about 100 products.<sup>1</sup> For demand models where the number of parameters grows with the number of product characteristics rather than the number of products, one might expect a small number of markets with a large number of products to give good estimates of demand.

This paper uses asymptotic approximations where asymptotics are taken in the number of products per market to examine the behavior of IV estimators of demand in large market settings, with a focus on product characteristic based instruments. Since the BLP instruments are correlated with prices only through equilibrium markups, their validity in this setting depends crucially on the nature of competition in markets with many products. If the dependence of markups on characteristics of other products disappears as the number of products increases and does so quickly enough, the BLP instruments will lose power in large markets and estimates based on them will be inconsistent when asymptotics are taken with respect to the number of products per market. The results in this paper use the asymptotic behavior of equilibrium markups to determine when this is the case.

I find that, in certain cases, the dependence of prices on product characteristic instruments through markups disappears at a fast enough rate that the BLP instruments lead to inconsistent estimates when asymptotics are taken in the number of products per market. In particular, this is the case with the logit and random coefficients logit models when the number of products increases with the number of markets and products per firm fixed.<sup>2</sup>

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<sup>1</sup>In addition to BLP and other papers on the automobile industry (e.g. Petrin, 2002), examples of industries with data that fit this description include personal computers (e.g. Eizenberg, 2011; Bresnahan, Stern, and Trajtenberg, 1997), LCD televisions (e.g. Conlon, 2012) and other consumer goods sold at a national level. Of these papers, Bresnahan, Stern, and Trajtenberg (1997) use BLP instruments exclusively, while the others use a combination of BLP and cost instruments.

<sup>2</sup>The random coefficients model considered in this paper does not allow a random coefficient on price.

Despite this negative result, I show that, in other settings, the dependence of markups on product characteristics remains or decreases slowly enough that the BLP instruments lead to consistent estimates under large market asymptotics. Under certain conditions, the BLP instruments lead to consistent estimates under logit demand when the number of products increases with the number of firms fixed, so long as firms are asymmetric. The asymptotic dependence of the markup on product characteristics can also be obtained when the number of products increases with the number of products per firm fixed if the dimension of the space of observed product characteristics increases with the number of products (but with enough restrictions that the dimension of the parameter space does not). I illustrate this point with a nested logit model with many nests. Finally, considering the case with many large markets, the BLP instruments will lead to consistent estimates in certain cases if the number of markets increases quickly enough relative to the number of products in each market.

This paper gives a negative result for any setting where (1) the number of products increases with the number of markets fixed, (2) the correlation between markups and characteristics of other products decreases quickly enough as the number of products increases under the researcher’s supply and demand specification (see Theorem 1 for a formal statement of “quickly enough”), (3) the researcher’s supply specification is correct, or needs to be correct for the analysis to go through, (4) cost instruments or other sources of identification are unavailable or insufficient to identify the model alone and (5) estimates of demand parameters are needed. If all of these conditions hold, IV estimates of the parameters of interest will be inconsistent. Regarding (4), instruments that shift marginal cost directly do not need variation in the markup to shift prices, and therefore do not suffer from the issues brought up here.

Regarding (3), if a particular supply specification is needed to estimate counterfactuals (e.g. prices after a merger), the analysis will rely on having both accurate demand estimates and a correct supply specification. If BLP instruments give poorly behaved demand estimates under a given supply specification, then these conditions cannot both be satisfied. However, certain applications of demand estimates do not require knowing the supply side specification (e.g. consumer welfare under the current regime). In these cases, specifying a particular model of supply that is consistent with BLP instruments having power will be less important, so long as there exists some plausible supply model that would lead to good performance of the BLP instruments.

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Allowing a random coefficient on price appears to require nontrivial extensions of the methods used in this paper, and is an important topic for future research.

While this paper draws on the literature on weak instruments<sup>3</sup> (see, for example, Stock, Wright, and Yogo, 2002), the main issue that this paper brings up is distinct from the problems with bias and size distortion considered in that literature, and cannot be remedied by methods proposed in that literature alone. The main issue is that the supply side model may constrain product characteristic instruments to have very low correlation with price. If this is the case and a researcher performs a test for identification and finds that these instruments are highly correlated with price, this must also be taken as evidence that the model is misspecified in ways that will likely lead to incorrect conclusions in policy counterfactuals based on the supply side model.<sup>4</sup> Performing specification tests based on supply side moments can help in diagnosing these issues, but these still have the possibility of giving the wrong answer because of sampling error.

In applications where the supply side is used in policy counterfactuals, the most thorough way of guarding against the issues brought up in this paper is to confirm that it is possible for BLP instruments to perform well when prices are generated from the supply side model being used elsewhere in the analysis and primitives are generated from a reasonable data generating process. This can be done through asymptotic approximations and Monte Carlo before even looking at the data. See section 4 for further discussion of how these issues can be diagnosed.

It should also be emphasized that, while this paper focuses on asymptotic approximations for their generality and tractability, the fundamental issue that the supply side may constrain product characteristics to perform poorly as price instruments is not a mere product of asymptotics, and can be elucidated in a finite sample framework through Monte Carlo. For a given sample size and data generating process for the model primitives, one can generate one data set with prices from a Bertrand equilibrium and another data set with markups set to a constant. If BLP instruments do not perform noticeably better in the first case (on average over Monte Carlo repetitions), one should look for different instruments or a different model of supply and demand. See section D.3 for an illustration of this approach.

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<sup>3</sup>More precisely, this paper derives weak instrument asymptotics as an equilibrium outcome of sequences of pricing games. See Pinkse and Slade (2010) for a discussion of related phenomena in the context of spatial models.

<sup>4</sup>See section C.1 of the supplementary appendix for a formal statement. The same issue will arise if the researcher performs inference in some other weak instrument robust way. Unless the supply side is misspecified, any weak instrument robust test will reject with probability close to its size asymptotically under any parameter value.

## 1.1 Related Literature

To my knowledge, the only other paper that considers asymptotics in the number of products per market in this setting is Berry, Linton, and Pakes (2004). In order to focus on other questions involving error from simulation based estimators and sampling error in product shares, those authors abstract from identification issues and from the supply side by placing high level conditions that assume that the instruments strongly identify the model under large market asymptotics. In contrast, the present paper asks which models of supply and demand allow product characteristic instruments to have power in a large market setting (and abstracts from the questions of simulation error and sampling error in market shares). In the cases where the present paper gives a negative answer to this question, the high level conditions for strong identification in Berry, Linton, and Pakes (2004) do not hold, and the present paper takes the further step of showing that this leads to inconsistent estimates.

In addition, there has been a recent literature proposing computational improvements and bias corrections in random coefficients demand models. Dubé, Fox, and Su (2012) and Conlon (2013) propose improved methods for computing these estimators, with the latter considering a generalized empirical likelihood estimator with improved higher order bias properties. Freyberger (2012) derives corrections for simulation error in the case where the number of markets is large relative to products per market.

More broadly, others have proposed different approaches to modeling and estimating demand in large markets, including Bajari and Benkard (2005) and Pinkse and Slade (2004). While the present paper focuses on the demand models and estimators proposed in BLP, one could make a similar point about taking into account the implications of an equilibrium model for the behavior of estimators in other settings where one deals with a small number of distinct venues in which many agents interact. Performing this type of analysis in other settings is expected to be a useful topic for future research.

This paper is also related to the literature on weak instruments (see Stock, Wright, and Yogo, 2002, for a survey of this literature). That literature uses sequences of underlying distributions in which the correlation of instruments with endogenous variables shrinks with the sample size to get asymptotic approximations that better approximate finite sample distributions. This paper shows that such sequences arise endogenously from equilibrium prices when asymptotics are taken in the number of products per market in a certain class of models. Other settings in which such sequences arise naturally have been observed in the literature on spatial econometrics (see Pinkse and Slade, 2010).

While the present paper focuses on product characteristic instruments, other papers have

dealt with the optimal use of other instruments when they are available, such as variables that shift marginal cost directly. In contemporaneous work, Reynaert and Verboven (2012) focus on settings where cost instruments are available and consider approximations to optimal functions of cost shifters and product characteristics based on the assumption of perfect competition (a setting where BLP instruments have no power). Their focus also differs from the present paper in that they focus on estimation of the distribution of the random coefficients ( $\sigma$  in the notation below), while the present paper focuses on the price parameter ( $\alpha$  in the notation below). Romeo (2010) proposes other instruments for models similar to those considered here.

In addition to the literatures on weak instruments and on estimation of discrete choice models of demand, this paper relates to theoretical results on oligopoly pricing in markets where demand is characterized by a discrete choice model. Gabaix, Laibson, Li, Li, Resnick, and de Vries (2013) consider the limiting behavior of prices in large markets in a similar setting to the present paper, but focus on different questions, leading to a different formulation and different results (for example, Gabaix, Laibson, Li, Li, Resnick, and de Vries achieve more generality in other directions by restricting attention to symmetric firms, while the present paper deals with instruments that attempt to exploit observed asymmetry between firms). Existence of equilibrium in some of the pricing games I consider follows from arguments in Caplin and Nalebuff (1991), Vives (2001) and Konovalov and Sandor (2010) or similar methods. There is also a literature examining how restrictions on demand elasticities in discrete choice models place restrictions on the possible outcomes of empirical applications (see, among others, Bajari and Benkard, 2003; Akerberg and Rysman, 2005). While some of the findings of this paper add to this body of literature, the main focus is on implications for the identifying strength of BLP instruments.

## 1.2 Plan for Paper

The paper is organized as follows. Section 2 describes the class of models being studied. Section 3 derives the asymptotic behavior of equilibrium prices and IV estimates in some commonly used models of supply and demand. Section 4 presents recommendations for diagnosing whether the supply side constrains BLP instruments to have poor power. Section 5 provides a Monte Carlo study. Section 6 concludes. Proofs and additional results are in a supplementary appendix.

## 2 The Model

This section describes the class of models and estimators considered in this paper and defines some notation that will be used later. The models and much of the notation follow BLP and Berry (1994).

The researcher observes data from a single market with  $J$  products labeled 1 through  $J$  and an outside good labeled 0, and  $M$  consumers. Each product  $j$  has a price  $p_j$ , a vector of other characteristics observed to the researcher,  $x_j \in \mathbb{R}^K$ , and an unobserved variable  $\xi_j$ , which can be interpreted as a combination of unobserved product characteristics and aggregate preference shocks. In addition, each individual consumer  $i$  has consumer specific unobserved components of demand  $\varepsilon_{ij}$  and  $\zeta_i$ , which are iid across consumers. In what follows,  $x_j$  is assumed to contain a constant.

Consumer  $i$ 's utility for the  $j$ th product is given by  $u_{ij} = u(x_j, p_j, \xi_j, \varepsilon_{ij}, \zeta_i)$  for some function  $u$ . Each consumer buys the product for which utility is the highest, and no consumer buys more than one product. Rather than individual purchasing decisions, we observe aggregate market shares, including the proportion of consumers who make no purchase (the share of the outside good). These come from aggregating purchasing decisions over the  $\varepsilon$ s and  $\zeta$ s of all consumers. It is assumed that the number of consumers is large enough that sampling variation in market shares from realizations of the  $\varepsilon$ s and  $\zeta$ s can be ignored, so that the market share of good  $j$ ,  $s_j(x, \xi, p)$ , is equal to the population probability of choosing good  $j$  conditional on  $x$ ,  $\xi$ , and  $p$ :  $s_j(x, \xi, p) = E_{\varepsilon, \zeta} I(u_{ij} > u_{ik} \text{ all } k \neq j)$ , where  $E_{\varepsilon, \zeta}$  denotes expectation with respect to the distribution of  $\{\varepsilon_{ij}\}_{j=1}^J$  and  $\zeta_i$ .

In the models considered here, utility can be written in the following form for some parameters  $(\alpha, \beta, \sigma)$ :  $u_{ij} = x'_j \beta - \alpha p_j + \xi_j + g(\varepsilon_{ij}, \zeta_i, x_j, p_j)$ , where  $\{\varepsilon_{i,j}\}_{j=1}^J$  are independent of  $\zeta_i$ , and the distribution of  $\zeta_i$  is indexed by a parameter  $\sigma$ . The linear part is denoted by  $\delta_j \equiv x'_j \beta - \alpha p_j + \xi_j$ . Since shares depend on  $\xi$ ,  $\alpha$  and  $\beta$  only through  $\delta$ , we can write them as  $s_j(\delta, x, p, \sigma)$ .

While some of the results in this paper use high level conditions on the markup that can apply more generally (see Theorem 1 below), most of this paper focuses on the static Bertrand supply model. There are  $F$  firms labeled 1 through  $F$ . Firm  $f$  produces the set of goods  $\mathcal{F}_f \subseteq \{1, \dots, J\}$ . Profits of firm  $f$  are given by  $\sum_{k \in \mathcal{F}_f} p_k \cdot M s_k(x, p, \xi) - C_f(\{M \cdot s_k(x, p, \xi)\}_{k \in \mathcal{F}_f})$  where  $M$  is the number of consumers and  $C_f$  is firm  $f$ 's cost function. Firms play a Nash-Bertrand equilibrium in prices, and rearranging the first order conditions for an interior best

response gives

$$\sum_{k \in \mathcal{F}_f} (p_k - MC_k) \frac{\partial}{\partial p_j} s_k(x, p, \xi) + s_j(x, p, \xi) = 0 \quad (1)$$

for each product  $j$  owned by firm  $f$ .<sup>5</sup> For single product firms, this simplifies to

$$p_j = MC_j - \frac{s_j(x, p, \xi)}{\frac{d}{dp_j} s_j(x, p, \xi)}. \quad (2)$$

When new products are added to the demand system, the equilibrium price will change, so that the equilibrium price and share of good  $j$  depend on the size of the market  $J$ . That is, even though  $x$ ,  $MC$ , and  $\xi$  are sequences, prices, markups and market shares will be triangular arrays, so that a more precise notation for the equilibrium price of good  $j$  would be  $p_{j,J}$  (in some cases, the instruments  $z_j$  defined below will change with  $J$  as well). To avoid extra subscripts, I use  $p_j$  to denote the price of good  $j$  when the context is clear.<sup>6</sup>

Finally, in all of the models below, I assume that the vector of unobserved demand shocks  $\xi$  is mean independent of observed product characteristics:  $E(\xi|x) = 0$ . This assumption is the exclusion restriction that provides the basis for the BLP instruments.

## 2.1 Estimation

In the models considered here,  $s(\delta, x, p, \sigma)$  is invertible in its first argument (see Berry, 1994, BLP). Letting  $\delta(s, x, p, \sigma)$  denote the inverse with respect to the first argument, this leads to the equation

$$\delta_j(s, x, p, \sigma) = x'_j \beta - \alpha p_j + \xi_j, \quad (3)$$

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<sup>5</sup>While the models of sections 3.2, 3.3 and 3.4 have a unique equilibrium, results showing whether the pricing game in the random coefficients logit model has an equilibrium (or a unique one) in the general setting of section 3.1 are, to my knowledge, not available in the literature. The results in that section hold for any sequence of equilibria so long as such a sequence exists, and do not impose uniqueness.

<sup>6</sup>While the entry decision is not explicitly modeled here, one could think of the market size  $J$  as being endogenously determined by firms' decisions of whether to pay a fixed cost of entering the market. If the number of consumers  $M$  is large relative to the entry cost, more firms will enter, and one can think of asymptotics in  $J$  as arising from asymptotics in  $M$  in a two stage model with endogenous entry (note that this interpretation requires that the information structure of the entry game is such that the exogeneity assumptions on  $x$  and  $\xi$  described below hold conditional on entry, which can be ensured by assuming that entry decisions are made before firms observe  $x$ ,  $\xi$  and marginal costs).



which can potentially be used to estimate the demand parameters  $\sigma$ ,  $\beta$  and  $\alpha$ . However, the parameter  $\sigma$  enters into a function with shares, which are endogenous. In addition, prices may be correlated with the unobserved  $\xi$  through at least two channels. First,  $\xi_j$  enters the markup  $-s(\delta, x, p, \sigma)/\frac{d}{dp_j}s(\delta, x, p, \sigma)$ . Second,  $\xi_j$  may be correlated with unobserved components of marginal costs if goods that are more desirable in unobserved ways are also more expensive to make in unobserved ways.

To overcome this endogeneity problem, one needs instruments that are uncorrelated with  $\xi$ , and shift prices enough to identify  $\alpha$  and  $\sigma$ . This paper focuses on instruments based on characteristics of other products. Since we are assuming that  $\xi_j$  is independent of the observed characteristics of all products, functions of characteristics of other products will satisfy the instrumental variables exclusion restriction. Since characteristics of other products enter price through the markup  $-s(\delta, x, p, \sigma)/\frac{d}{dp_j}s(\delta, x, p, \sigma)$ , they have the potential to shift prices enough to consistently estimate the model. Suppose that we use some vector valued function  $h_j(x_{-j})$  as excluded instruments. The parameter estimates minimize the GMM criterion function

$$\left\| \frac{1}{J} \sum_{j=1}^J (\delta_j(s, x, p, \sigma) - x'_j \beta + \alpha p_j) z_j \right\|_{W_J} \quad (4)$$

where  $z_j = (x_j, h_j(x_{-j}))'$  and  $W_J$  is a positive definite weighting matrix with  $W_J \xrightarrow{p} W$  for a strictly positive definite matrix  $W$ . Following common terminology, this paper refers to instruments of this form as product characteristic instruments or BLP instruments.

### 3 Large Market Asymptotics

I now turn to the question of the asymptotic behavior of demand estimates, particularly those based on product characteristic instruments, under large market asymptotics. I first state a general result relating the behavior of BLP instrument based estimates to the asymptotic behavior of the markup. The remainder of this section is organized into subsections that show consistency or inconsistency of BLP instrument based estimates for various settings using primitive conditions.

To give some motivation for the result, let us first consider a special case. Consider the simple logit model with many small firms in a single market. Consumer  $i$ 's utility for product  $j$  takes the form  $u_{i,j} = x'_{i,j} \beta - \alpha p_j + \varepsilon_{i,j} + \xi_j$  where  $\varepsilon_{i,j}$  is distributed extreme value independently across products and consumers. This leads to shares taking the form

$s_j(x, p, \xi) = \frac{\exp(x'_j \beta - \alpha p_j + \xi_j)}{\sum_k \exp(x'_k \beta - \alpha p_k + \xi_k)}$ , which can be inverted to get (normalizing the mean utility of the outside good 0 to zero)  $\log s_j - \log s_0 = x'_j \beta - \alpha p_j + \xi_j$ . The derivative of firm  $j$ 's share with respect to  $j$ 's price is  $\frac{d}{dp_j} s_j(x, p, \xi) = -\alpha s_j(x, p, \xi)(1 - s_j(x, p, \xi))$ , which gives the Bertrand pricing formula, equation (2), as  $p_j = MC_j + \frac{1}{\alpha(1-s_j)}$ .

As long as shares converge to zero, markups of all products will converge to  $1/\alpha$ . If the markup were exactly equal to  $1/\alpha$ , product characteristic instruments would yield inconsistent estimates, since they must be correlated with markups to have identifying power. If the convergence of the markup to  $1/\alpha$  is fast enough, one would expect this to be true for the actual sequence of markups. More generally, whenever the dependence of equilibrium markups on characteristics of other products decreases quickly enough with  $J$ , product characteristic instruments will lead to inconsistent estimates. The following theorem formalizes these ideas.<sup>7</sup>

**Theorem 1.** *Let  $(x_j, \xi_j, MC_j)$  be iid with finite second moment, and let  $(\hat{\alpha}, \hat{\beta}')$  be the IV estimates with  $\sigma$  fixed at its true value and instrument vector  $z_j = (x_j, h_j(x_{-j}))$ . Suppose that*

(i)  $\sqrt{J} \max_{1 \leq j \leq J} |p_j - MC_j - b^*| \xrightarrow{P} 0$  for some constant  $b^*$ .

(ii)  $\frac{1}{\sqrt{J}} \sum_{j=1}^J [z_j(x'_j, MC_j, \xi_j) - E z_j(x'_j, MC_j, \xi_j)]$  converges to a nondegenerate normal distribution and  $\frac{1}{J} \sum_{j=1}^J E h_j(x_{-j})$  converges to some finite constant as  $J \rightarrow \infty$ .

Let  $(\hat{\alpha}^*, \hat{\beta}^{*'})$  be the same estimates obtained from data with  $p_j$  replaced by  $p_j^* = MC_j + b^*$ . Then  $(\hat{\alpha}, \hat{\beta}')$  is inconsistent and  $\|(\hat{\alpha}, \hat{\beta}') - (\hat{\alpha}^*, \hat{\beta}^{*'})\| \xrightarrow{P} 0$ .

Theorem 1 states that, as long as markups converge to a constant at a faster than  $\sqrt{J}$  rate, BLP instruments will lead to inconsistent estimates, even if  $\sigma$  is known and used in estimation. The conditions on markups are given as high level conditions, so that Theorem 1 does not require the specific structure of any of the demand specifications, supply models, or equilibrium assumptions used below. As long as the dependence of equilibrium markups on product characteristics decreases at a faster than  $\sqrt{J}$  rate, BLP instruments will give

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<sup>7</sup>This theorem, which is used to derive inconsistency results in the random coefficients logit model in section 3.1, applies to  $(\alpha, \beta)$  with the parameter  $\sigma$  determining the random coefficients treated as known. Since the BLP instruments lead to inconsistent estimates in that setting, the (negative) message is essentially the same: even with  $\sigma$  known, product characteristic instruments do not give consistent estimates. However, extending consistency results such as those in sections 3.2, 3.3 and 3.4 to more general specifications of random coefficients is left for future research.

inconsistent estimates.<sup>8</sup> In the simple logit model with single product firms, markups are given by  $\frac{1}{\alpha(1-s_j)}$ , which can be seen to converge to  $1/\alpha$  at a  $1/J$  rate as long as all of the market shares are roughly proportional to each other. The  $1/J$  rate is fast enough to lead to inconsistent estimates by Theorem 1.

Section 3.1 gives a formal statement of these results in the random coefficients logit model, which generalizes the simple logit model used in the discussion above. Sections 3.2, 3.3 and 3.4 consider cases where the dependence of markups on product characteristics does not decrease or decreases slowly enough that BLP instruments have identifying power asymptotically. Section B in the supplementary appendix considers some other cases. In all of these cases, the unifying feature that determines whether the BLP instruments can lead to consistent estimates is whether the dependence of markups on product characteristics decreases at a slower rate than the square root of the total number of products.

### 3.1 Random Coefficients Logit with Single Product Firms

The random coefficients logit model, used by BLP, generalizes the simple logit, allowing for more general forms of consumer heterogeneity through random coefficients on the observed product characteristics  $x$ . Consumer  $i$ 's utility for product  $j$  takes the form

$$u_{ij} = x'_{j}\beta - \alpha p_j + \xi_j + \sum_k x_{jk}\zeta_{ik} + \varepsilon_{ij} \equiv \delta_j + \sum_k x_{jk}\zeta_{ik} + \varepsilon_{ij}$$

where  $\zeta_{ik}$  is a random coefficient on product  $k$  and  $\varepsilon_{i,j}$  is distributed extreme value independently across products and consumers. This specification assumes that there is no random coefficient on price. In practice, including a random coefficient on price can be important because of differences in price sensitivity among consumers. While it seems plausible that similar results will hold with a random coefficient on price, determining whether this is the case appears to require a nontrivial extension of the results below that does not yield to the same method of proof.

This section considers asymptotics in which the number of products increases with a single product per firm. The single product firm assumption is made for simplicity, and the results will be similar as long as products are added by increasing the number of firms rather than the number of products per firm (see section B.3 of the supplementary appendix). It is

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<sup>8</sup>In the case with  $N$  markets  $i = 1, \dots, N$  with  $J_i$  products in market  $i$  and  $N \rightarrow \infty$ , condition (i) can be modified by replacing  $J$  with  $\sum_{i=1}^N J_i$ , and with  $b^*$  constant across both  $i$  and  $j$ , which leads to inconsistency when markups approach a constant more quickly than  $\sqrt{\sum_{i=1}^N J_i}$  (see section 3.4).

also assumed that the dimension of the random coefficients  $\zeta$  is fixed, and that new products differ only in drawing new characteristics ( $x_j$  and  $\xi_j$ ) and  $\varepsilon_{i,j}$  terms. As shown in sections 3.2 and 3.3, increasing the dimension of the random coefficients or the number of products per firm can lead to dramatically different results.

Shares can be obtained by integrating the logit shares for fixed  $\zeta$ , which gives, letting  $P_\zeta$  be the probability measure of the random coefficients,  $s_j = \int \tilde{s}_j(\delta, \zeta) dP_\zeta(\zeta)$  where  $\tilde{s}_j(\delta, \zeta) = \frac{\exp(\delta_j + \sum_k x_{jk}\zeta_k)}{\sum_\ell \exp(\delta_\ell + \sum_k x_{\ell k}\zeta_k)}$ . Differentiating under the integral and using the formulas for logit elasticities for fixed  $\zeta$  gives  $\frac{ds_j}{dp_j} = -\alpha \int \tilde{s}_j(\delta, \zeta)(1 - \tilde{s}_j(\delta, \zeta)) dP_\zeta(\zeta)$  so that the Bertrand markup is  $p_j - MC_j = \frac{\int \tilde{s}_j(\delta, \zeta) dP_\zeta(\zeta)}{\alpha \int \tilde{s}_j(\delta, \zeta)(1 - \tilde{s}_j(\delta, \zeta)) dP_\zeta(\zeta)}$ . If the tails of  $\zeta$  are thin enough, this can be shown to approach  $1/\alpha$  quickly by truncating the integral and using bounds on the logit shares for  $\zeta$  fixed, then arguing as in the simple logit model.

**Theorem 2.** *In the random coefficients model with single product firms and no random coefficient on price, suppose that  $(x_j, \xi_j, MC_j)$  is bounded and iid over  $j$ , and that the tails of the distribution of  $\zeta$  are bounded by the tails of a normal random variable. Then, for  $p_j$  arising from any sequence of Bertrand equilibria, condition (i) in Theorem 1 holds with  $b^* = 1/\alpha$ . Thus, under condition (ii) in Theorem 1, the BLP instrument based estimator with  $\sigma$  known will lead to inconsistent estimates.*

Theorem 2 shows that product characteristics lead to inconsistent estimates of  $\alpha$  and  $\beta$  even if the nonlinear parameter  $\sigma$  is known and used in estimation. The boundedness condition on  $(x_j, \xi_j, MC_j)$  is imposed for simplicity, and can be replaced by an exponential tail condition (see section A.2 in the supplementary appendix).

### 3.2 Nested Logit with Many Nests

The results of section 3.1 show that markups converge to a constant as  $J \rightarrow \infty$  when a new idiosyncratic term  $\varepsilon_{i,j}$  is added for each product, with the distribution of the random coefficients  $\zeta$  staying the same. One way of avoiding this negative result is to increase the dimension of  $\zeta$  as the number of products increases. Increasing the dimension of  $\zeta$  in a completely unrestricted way, one would end up with an increasing number of parameters, which leads to its own problems. Section B.1 of the supplementary appendix considers the nested logit model, a special case of the random coefficients logit model in which products are placed in groups, and the random coefficients are associated with group indicator variables. The nested logit model places enough structure on the random coefficients that the number of groups can be increased without increasing the number of parameters that need to be

estimated. The results in section B.1 show that under asymptotics where the dimension of  $\zeta$  is increased by adding more groups, BLP instruments can have power asymptotically even with single product firms.

It should be emphasized that the nested logit model is used for tractability and is intended to illustrate the point that one can reverse the negative results of section 3.1 through a specification where the dimension of the random coefficients increases with the number of products while the dimension of the parameter space stays fixed. One could likely achieve a similar goal through other specifications of random coefficients (for example, starting with the nested logit model with many nests and adding some random coefficients to continuous variables, as in Grigolon and Verboven, 2013), although such an approach will still require constraints on the joint distribution of random coefficients to keep the number of parameters from increasing.

### 3.3 Logit with Many-Product Firms

Now consider the logit model with multiproduct firms, with the number of firms  $F$  fixed and asymptotics taken in the number of products per firm. The own and cross price elasticities can be shown to take the form  $\frac{d}{dp_j} s_j(x, p, \xi) = -\alpha s_j(x, p, \xi)(1 - s_j(x, p, \xi))$  and  $\frac{d}{dp_k} s_j(x, p, \xi) = \alpha s_j(x, p, \xi) s_k(x, p, \xi)$  respectively. Plugging these into the equilibrium pricing equations (1) and rearranging gives the markup of product  $j$  produced by firm  $f$  as  $p_j - MC_j = \frac{1}{\alpha} + \sum_{k \in \mathcal{F}_f} (p_k - MC_k) s_k$ , so that markups are constant within a firm (see Konovalov and Sandor, 2010). Letting  $b_f$  be the common markup for firm  $f$ , this gives a system of equations that define  $b_f$  for  $f = 1, \dots, F$ . Rearranging and plugging in the formula for shares yields

$$b_f = \frac{1}{\alpha} \frac{\sum_{k \in \mathcal{F}_f} \exp(x'_k \beta - \alpha MC_k + \xi_k - \alpha b_f)}{\sum_{h \neq f} \sum_{k \in \mathcal{F}_h} \exp(x'_k \beta - \alpha MC_k + \xi_k - \alpha b_h)} = \frac{1}{\alpha} \frac{\exp(-\alpha b_f) \hat{\pi}_f \bar{r}_f}{\sum_{h \neq f} \exp(-\alpha b_h) \hat{\pi}_h \bar{r}_h} \quad (5)$$

where  $\hat{\pi}_f \equiv |\mathcal{F}_f|/J$  is the proportion of products produced by firm  $f$  and  $\bar{r}_f$  is an average of the characteristics of firm  $f$ 's products given by  $\bar{r}_f \equiv \frac{1}{|\mathcal{F}_f|} \sum_{k \in \mathcal{F}_f} \exp(x'_k \beta - \alpha MC_k + \xi_k)$ .

Under a law of large numbers,  $\bar{r}_f$  will converge to some  $\mu_{r,f}$  for each firm  $f$ . Assuming that  $\hat{\pi}_f$  also converges to some  $\pi_f$  for each firm  $f$ , this suggests that equilibrium prices will be determined asymptotically by the solution to equation (5) with  $\bar{r}_f$  and  $\hat{\pi}_f$  replaced by  $\mu_{r,f}$  and  $\pi_f$ . This is formalized in the following theorem.

**Theorem 3.** *In the simple logit model with asymptotics in the number of products per firm, suppose that  $(x_j, \xi_j, MC_j)$  is independent across all  $j$  and identically distributed within*

each firm with finite variance. Let  $z_j = \left(x_j, \frac{1}{J} \sum_{k \in \mathcal{F}_f} \tilde{h}(x_k)\right)$  for some function  $\tilde{h}$  for product  $j$  owned by firm  $f$ . Let  $\mu_{r,f} = E \exp(x'_j \beta - \alpha MC_j + \xi_j)$ ,  $\mu_{h,f} = E \tilde{h}(x_j)$  and  $V_f = E \begin{pmatrix} x_j \\ \pi_f \mu_{h,f} \end{pmatrix} (x'_j, \pi_f \mu'_{h,f}) \xi_j^2$  for  $j \in \mathcal{F}_f$  (where these quantities are assumed to be finite), and suppose  $\hat{\pi}_f \rightarrow \pi_f$  for some  $\pi_f$  for each  $f$ . Let  $(b_1^*, \dots, b_F^*)$  be the unique solution to (5) with  $\hat{\pi}_f \bar{r}_f$  replaced by  $\pi_f \mu_{r,f}$  for each firm  $f$ , and let  $p_j^* = MC_j + b_f^*$  for product  $j$  produced by firm  $f$ , and let  $(\hat{\alpha}, \hat{\beta}')$  be the estimators defined by (4) with these instruments, and let  $(\hat{\alpha}^*, \hat{\beta}^*)$  be defined in the same way, but with  $p_j^*$  replacing  $p_j$ . Then, if  $\frac{1}{J} \sum_{j=1}^J E z_j(x'_j, p_j^*) \rightarrow M_{zx}$  for a positive definite matrix  $M_{zx}$ ,

$$\sqrt{J}[(\hat{\beta}', -\hat{\alpha})' - (\beta', -\alpha)'] \xrightarrow{d} N \left( 0, (M'_{zx} W M_{zx})^{-1} M'_{zx} W \left( \sum_{f=1}^F \pi_f V_f \right) W M_{zx} (M'_{zx} W M_{zx})^{-1} \right),$$

and the same holds for  $(\hat{\beta}^*, -\hat{\alpha}^*)$ .

Theorem 3 shows that product characteristic instruments can have identifying power in this setting if they exploit variation in  $\pi_f \mu_{r,f}$  across firms. Thus, BLP instruments can have power through variation across firms, but not within firms.

### 3.4 Many Large Markets

According to the results of section 3.1, the BLP instruments lose power in a single market (or any fixed number of markets) with many firms fast enough that estimates based on them are inconsistent. In contrast, BLP instruments will typically provide enough variation to consistently estimate these models if the market size is bounded, and asymptotics are taken in the number of markets. This section considers the intermediate case where both the number of products and markets are allowed to go to infinity. To simplify the analysis, attention is restricted to the simple logit model with no random coefficients.

Some additional notation is needed to describe the results with many markets. We consider  $N$  markets, with  $J_i$  products in market  $i$ . Notation is otherwise the same as described in section 2, except that prices, product characteristics, etc. are now indexed by the market  $i$  as well as the product  $j$ , so that  $p_{i,j}$  denotes the price of product  $i$  in market  $j$  (as before, the dependence of  $p_{i,j}$  on the total market size  $J_i$  is suppressed in the notation). Let  $\bar{J} = \frac{1}{N} \sum_{i=1}^N J_i$  be the average number of products per market. Under the asymptotics in this section, each  $J_i$  (and therefore  $\bar{J}$ ) increases with the number of markets  $N$ , but the dependence of  $\bar{J}$  and the  $J_i$ s on  $N$  is suppressed in the notation.

Let  $v_N = \frac{1}{N} \sum_{i=1}^N (J_i - \bar{J})^2 / \bar{J}^2$  and  $m_3 = \frac{1}{N} \sum_{i=1}^N (J_i / \bar{J})^3$ . Here, the  $J_i$ s are nonrandom, but  $v_N$  can be thought of as the normalized sample variance of the  $J_i$ s. The following theorem derives the asymptotic behavior of estimates based on the BLP instruments in the case where  $v_N$  is bounded away from zero.

**Theorem 4.** *In the simple logit model with many large markets, suppose that  $(x_{i,j}, \xi_{i,j}, MC_{i,j})$  is bounded and iid across  $i$  and  $j$ . Let  $(\hat{\alpha}, \hat{\beta})$  be IV estimates with instrument vector  $z_{i,j} = (x_{i,j}, \frac{1}{J} \sum_{k \neq j} \tilde{h}(x_{i,k}))$  for some finite variance function  $\tilde{h}$ . Suppose that  $\min_{1 \leq i \leq N} J_i \rightarrow \infty$  and that  $v_N \rightarrow v$  for some  $v > 0$  and  $m_3$  converges to a finite constant. Then, if  $N/\bar{J} \rightarrow \infty$ ,  $(\hat{\alpha}, \hat{\beta})$  will be consistent and asymptotically normal, with a  $\sqrt{N/\bar{J}}$  rate of convergence and asymptotic variance given in the supplementary appendix. If  $N/\bar{J} \rightarrow c$  for some  $c$ , then  $(\hat{\alpha}, \hat{\beta})$  will be inconsistent, and will follow a weak instrument asymptotic distribution given in the supplementary appendix. The weak instrument asymptotic distribution coincides with what would be obtained with markups equal to  $1/\alpha$  in the case where  $c = 0$ .*

Theorem 4 states that, as long as there is sufficient variation in the number of products per market, the BLP instruments will use this variation to obtain consistent estimates at a rate  $\sqrt{N/\bar{J}}$  as long as  $N/\bar{J} \rightarrow \infty$ . If  $N/\bar{J} \rightarrow c$  for some finite  $c$ , the results give “weak instrument” asymptotics, in which the estimates do not converge and follow a nonstandard asymptotic distribution. The dependence on whether  $N/\bar{J} \rightarrow \infty$  comes from an extension of condition (i) in Theorem 1 to the many market case. In general, BLP instruments require that the markup not converge to a constant more quickly than  $\sqrt{N/\bar{J}}$ . Since the logit markups in market  $i$  converge to  $1/\alpha$  at a  $1/J_i$  rate, this means that the estimates will be inconsistent if  $\sqrt{N/\bar{J}}/J_i$  converges to zero uniformly over  $i$  which (assuming  $\bar{J}/J_i$  is bounded), gives the  $N/\bar{J} \rightarrow 0$  condition for inconsistency and asymptotic equivalence with constant markups. Note that Theorem 4 shows that the identifying power of BLP instruments in this setting relies on variation in market size ( $v_N$  must not converge to zero). If one is not comfortable using this variation to identify demand (for example, because of issues raised by Akerberg and Rysman, 2005), one will want to look for other instruments or a different specification.

The assumption that the data generating process for marginal costs is the same across markets includes an important assumption about how the cost function varies with market size. If marginal cost varies systematically with  $J_i$  (depending on how  $J_i$  varies with the total number of consumers in a market, this could arise from returns to scale), one can use  $J_i$  or  $\sum_{k \neq j} \tilde{h}(x_{i,k})$  as cost instruments, whether or not they are correlated with markups. It should also be noted that, while Theorem 4 gives the rate at which the  $J_i$ 's must increase

when  $\frac{1}{J} \sum_{k \neq j} \tilde{h}(x_{i,k})$  is used as the excluded instrument, other forms of BLP instruments (i.e. other functions of  $\{x_{i,k}\}_{k=1}^{J_i}$ ) may lead to consistent estimates under weaker conditions.

## 4 Diagnosis and Recommendations for Empirical Practice

Theorem 1 and the results in section 3.1 state that certain specifications of supply and demand constrain BLP instruments to perform poorly under certain asymptotics. On the other hand, the results in sections 3.2, 3.3 and 3.4 show that more positive results are possible in essentially the same model, depending on the relative rate at which the number of products, firms and markets increase. This raises the question of how one can determine which asymptotics are relevant for a given specification and data set and, in particular, whether one should worry that the asymptotic results of Theorem 1 are relevant. This section discusses three ways of addressing this issue: a priori verification of internal consistency of supply with identification of demand (discussed in section 4.1) tests for identification (discussed in section 4.2) and tests for misspecification (discussed in section 4.3).

Which combination of these methods is appropriate will depend on whether correct specification of the supply side is necessary for the overall analysis. If the supply side needs to be specified correctly (e.g. for computing prices after a merger), the most thorough way of guarding against the negative results in this paper is to determine, before even looking at the data, that it is possible to get accurate demand estimates using BLP instruments when prices are generated from the specified supply and demand model. This can be done using the asymptotic approximations in section 3 along with Monte Carlo analysis, as discussed in section 4.1 below. Such an approach is preferable to one based solely on statistical tests, since it does not run the risk of giving the wrong answer due to sampling error, and because it does not suffer from issues with pre-testing (see section 4.4 for a discussion of the latter issue). However, if computational limitations make such a Monte Carlo analysis prohibitive, the tests for identification and misspecification in sections 4.2 and 4.3 can be used.

For other applications (e.g. computing welfare under the observed equilibrium), specifying a supply side is not necessary. If the researcher does not specify a supply model, the diagnoses in sections 4.1 and 4.3 will not be applicable (since they use a supply specification given by the researcher). In this case, the recommendations in section 4.2 may suffice. However, to the extent that the analysis will be more plausible if the researcher can exhibit a supply model that leads to demand estimates performing well, the recommendations in



sections 4.1 and 4.3 will be useful as well.

## 4.1 Monte Carlo Verification of Internal Consistency of Supply with Identification of Demand

The asymptotic results in section 3 can be used as a guide to specifying a model of demand that is internally consistent with BLP instruments having identifying strength with a given model of supply. To examine these issues in finite samples for a particular data set, one can perform a Monte Carlo analysis to see whether it is possible to obtain accurate estimates of demand (or tests with a good size-power tradeoff) when prices are generated from the full supply and demand model with a plausible data generating process for model primitives. If the estimates and tests based on BLP instruments perform well enough in the Monte Carlos with prices generated from the full model, the researcher can conclude that the negative results of Theorem 1 and section 3.1 do not apply.

An advantage of this approach is that it does not use the data to decide how to proceed with the analysis (or, indeed, whether to proceed at all). Thus, it does not suffer from the issues with pre-testing (described in section 4.4 below) associated with tests for identification and specification.

## 4.2 Tests for Identification

Under the conditions of Theorem 1, which lead to inconsistent estimates, the coefficients of the BLP instruments  $h_j(x_{-j})$  in the regression of prices  $p_j$  on own product characteristics  $x_j$  and instruments  $h_j(x_{-j})$  will converge in probability to zero. Thus, one way of testing whether the conditions of this negative result accurately describe a given data generating process is to run an OLS regression of  $p_j$  on  $(x_j, h_j(x_{-j}))$ , and perform an  $F$  test of the null hypothesis that the coefficients on  $h_j(x_{-j})$  are zero. If the test rejects, one can conclude that the correlation of the instruments with prices in the data is large enough that the negative results of Theorem 1 are not an accurate description of the data generating process. See section C.1 for a formal statement and details of the test.

The test described above corresponds to a test of the null of lack of identification in a linear model with a single endogenous variable. While this provides a valid test of the null hypothesis that the conditions of Theorem 1 hold, it does not provide a test of the more general null hypothesis of lack of identification when the nonlinear parameter  $\sigma$  is unknown. See Wright (2003) for tests of the general null hypothesis of lack of identification.

### 4.3 Tests of Overidentifying Restrictions

If the identification test from the previous section rejects, one can conclude that the conditions of Theorem 1 do not match the correlation between prices and instruments in the data. However, the negative results of Theorem 1 will still be relevant in this case if the researcher computes counterfactual outcomes using a supply and demand model that satisfies the conditions of this theorem. If this is the case, rejecting with the test in the previous section will be the result of misspecification.

For a given supply model, one can test for this form of misspecification as follows. Let  $b_{i,j}(\alpha, \beta, \sigma) = b_{i,j}(p_i, x_i, \xi_j(\alpha, \beta, \sigma, x_i, p_i); \alpha, \beta, \sigma)$  be the markup for product  $j$  in market  $i$  recovered from the data assuming the parameter values  $(\alpha, \beta, \sigma)$  and the researcher's supply model (for the static Bertrand model used in most of this paper, this corresponds to equation (3.5) in BLP; however,  $b_{i,j}$  can be defined using other supply models as well). Consider the GMM moment conditions formed by stacking the moment conditions for (4) with the supply side moments

$$E(p_{i,j} - b_{i,j}(\alpha, \beta, \sigma) - \gamma' x_{i,j}) \begin{pmatrix} x_{i,j} \\ h_j(x_{i,-j}) \end{pmatrix} = 0 \quad (6)$$

If the supply side model used to compute  $b_{i,j}$  matches the data generating process, and if  $h_j(x_{i,-j})$  does not enter marginal cost directly (i.e. it is a BLP instrument rather than a cost instrument), (6) will hold. A test for misspecification of the supply side can be obtained by stacking these supply side moments with the demand side moments and performing a standard GMM test of overidentifying restrictions.

Section C.2 in the supplementary appendix provides the details of this test and shows formally that this test has power to detect cases where the test in section 4.2 rejects the conditions of Theorem 1 with high probability and the model used to compute  $b_{i,j}$  satisfies the conditions of Theorem 1. To get some intuition for this result, let  $\hat{\pi}_p$  be the coefficient on  $h_j(x_{i,-j})$  in a regression of  $p_{i,j}$  on  $x_{i,j}$  and  $h_j(x_{i,-j})$  (so that the test in section 4.2 rejects when  $\hat{\pi}_p$  is far from zero). Let  $\hat{\pi}_{\text{markup}}$  be the coefficient on  $h_j(x_{i,-j})$  in a regression of  $b_{i,j}(\hat{\alpha}, \hat{\beta}, \hat{\sigma})$  on  $x_{i,j}$  and  $h_j(x_{i,-j})$ , and let  $\hat{\gamma}_h$  be the coefficient on  $h_j(x_{i,-j})$  in a regression of  $p_{i,j} - b_{i,j}(\hat{\alpha}, \hat{\beta}, \hat{\sigma})$  on  $x_{i,j}$  and  $h_j(x_{i,-j})$  (so that (6) implies that the population counterpart to  $\hat{\gamma}_h$  is zero). By linearity of OLS formula, the relation

$$\hat{\pi}_p = \hat{\gamma}_h + \hat{\pi}_{\text{markup}} \quad (7)$$

holds in finite samples. If the BLP instruments have power in a correctly specified model, they will be correlated with the markup, so that  $\hat{\pi}_{\text{markup}}$  will be far from zero. According to (7), one can check whether this is the case by making sure that  $\hat{\pi}_p$  is far from zero (i.e. checking that lack of identification is rejected with the test in section 4.2) and that  $\hat{\gamma}_h$  is close to zero (i.e. making sure that a specification test based on  $\hat{\gamma}_h$  does not lead to rejection).

#### 4.4 A Warning Regarding Pre-Testing

The tests described in sections 4.2 and 4.3 can be useful in diagnosing whether certain issues brought up in this paper are relevant in a given data set. However, if one uses the results of these tests to decide on a specification (e.g. if one uses these tests to decide on cost instruments vs BLP instruments, or, indeed, whether to proceed at all), the overall procedure may suffer from undercoverage. While weak instrument robust procedures (see, e.g., Kleibergen, 2005) can be used to guard against size distortion in settings like the ones in this paper, the same criticism applies when these tests are embedded in a procedure where one chooses the specification based on the data. Note that this critique applies not only to BLP instruments, but to any setting where first stage tests are used to decide which IV specification to report.

Since the analysis suggested in section 4.1 uses only theoretical insights and Monte Carlos, it can be performed without looking at the data (except to calibrate sample size, etc.). Thus, it does not suffer from the pre-testing issues described above: deciding on a specification (or even whether to proceed at all) based on such an analysis will not result in size distortions. In this regard, the fact that the performance of BLP instruments is closely tied to theoretical supply and demand models is an advantage of these instruments in settings where an explicit supply side model is available. See Chioda and Jansson (2005) and Andrews (2014) for further discussion of pre-testing issues, with particular reference to IV models.

## 5 Monte Carlo

This section presents the results of a Monte Carlo study of the random coefficients logit model of section 3.1. The performance of the BLP instruments and of cost instruments is examined over a range of specifications for the number of markets, the number of products per market and variation in the number of products per market. The data generating process for the Monte Carlo data sets is as follows. Prices are generated from a Bertrand equilibrium. For the case where the number of products per market varies, approximately 1/3 of markets

have 20 products, another 1/3 have 60 products, and the remaining have 100 products. For the case where the number of products per firm varies, approximately 1/3 of markets have 2 products per firm, another 1/3 have 5 products per firm, and the remaining have 10 products per firm.  $x_{i,j}$  contains a constant and a uniform  $(0, 1)$  random variable. I generate the cost shifter,  $z_{i,j}$ , as another uniform random variable independent of  $x_{i,j}$ . Marginal cost is given by  $MC_{i,j} = (x'_{i,j}, z'_{i,j})\gamma + \eta_{i,j}$  for  $\eta_{i,j}$  defined as follows. To generate  $\eta$  and  $\xi$ , I generate three independent uniform  $(0, 1)$  random variables  $u_{1,i,j}$ ,  $u_{2,i,j}$ , and  $u_{3,i,j}$ , and set  $\xi_{i,j} = u_{1,i,j} + u_{3,i,j} - 1$  and  $\eta_{i,j} = u_{1,i,j} + u_{2,i,j} - 1$ .  $x_{i,j}$ ,  $\xi_{i,j}$ , and  $\eta_{i,j}$  are independent across products  $j$ . Utility is given by the random coefficients model of section 3.1, with the random coefficient on the covariate given by a  $N(0, \sigma^2)$  random variable, where  $\sigma^2$  is set to 9 and is estimated in the Monte Carlo. The parameters are given by  $\alpha = 1$  and  $\gamma = (2, 1, 1)'$  (where the last element of  $\gamma$  is the coefficient of the excluded cost instrument), with  $\beta$  taking different values depending on the design. See the supplementary appendix for additional details, and results for additional specifications.<sup>9</sup>

Tables 1, 2 and 3 show the results for BLP instruments and cost instruments for several Monte Carlo designs. Results are reported for estimates of the price coefficient  $\alpha$  and for a nominal level .05 two sided test for  $\alpha$ . The Monte Carlo results appear consistent with the overall result that BLP instruments perform poorly for this demand specification when the number of products is large enough relative to the number of markets, and that cost instruments do not suffer from these issues. In Table 1, estimates that use BLP instruments have substantial bias and variability when the number of products per market is large and the number of markets small (as measured by median bias and median absolute deviation from the true value), and perform better when the number of products per market is small. In contrast, cost instruments lead to relatively precise estimates in large market settings. Table 2 fixes the number of products and markets and explores how variation in other aspects of the design affects the performance of the BLP instruments. While the BLP instruments work well in some cases, the results can be very bad depending on the ownership structure and the coefficient of the product characteristic in the demand specification.

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<sup>9</sup>See also the contemporaneous work of Skrainka (2012) and Conlon (2013) for additional Monte Carlo results for BLP and cost instruments with prices generated from equilibrium play.

markets	products per market	median bias of $\hat{\alpha}$	median absolute deviation from $\alpha_0$	rejection prob. at true $\alpha$	power of test of $\alpha = 0$
1	60	-0.3698	0.6691	0.0783	0.1004
1	100	-0.3648	0.7177	0.1211	0.1381
3	20	-0.1053	0.3358	0.0390	0.3980
3	varied	-0.0494	0.2966	0.1523	0.4649
3	60	-0.2186	0.5827	0.0200	0.1040
3	100	-0.2525	0.6383	0.1351	0.1762
20	20	0.0039	0.1140	0.0390	0.9880
20	varied	-0.0014	0.1001	0.0400	0.9960
20	60	0.0130	0.2345	0.0230	0.7710
20	100	-0.0379	0.3154	0.0200	0.4560

Table 1: Monte Carlo Results for BLP Instruments (10 Products per Firm,  $\beta = (3, 6)$ )

$\beta$	products per firm	median bias	median absolute deviation from $\alpha_0$	rejection prob. at true $\alpha$	power of test of $\alpha = 0$
(3,1)	10	-0.2928	0.8090	0.1012	0.1273
(3,1)	2	-0.3573	0.7653	0.0991	0.1241
(3,1)	varied	-0.0054	0.1895	0.0380	0.8440
(3,6)	2	-0.2191	0.6897	0.1061	0.1632
(3,6)	varied	-0.0190	0.1659	0.0410	0.9450

Table 2: Monte Carlo Results for BLP Instruments (20 Markets, 100 Products per Market)

## 6 Conclusion

This paper derives asymptotic approximations for differentiated products demand estimators when the number of products is large. The question of whether product characteristic instruments have nontrivial identifying power is addressed through asymptotic correlation with equilibrium markups derived from a full model of supply and demand. The results show that certain supply and demand models constrain these instruments to have trivial power under large market asymptotics, and should therefore be avoided in cases where these asymptotic results are relevant. Other asymptotic settings (demand models and ways of adding products) are shown to lead to consistent estimates and standard asymptotic distributions under large market asymptotics. The results can be used as a guide to forming a model of demand in a large market setting that is consistent with finding identification through product characteristic instruments and variation in the markups. A Monte Carlo study shows that the asymptotic results are a good description of finite sample settings of practical importance.

markets	products per market	median bias of $\hat{\alpha}$	median absolute deviation from $\alpha_0$	rejection prob. at true $\alpha$	power of test of $\alpha = 0$
1	60	-0.0247	0.1749	0.1130	0.6710
1	100	-0.0196	0.1358	0.0762	0.7623
3	20	-0.0262	0.1837	0.1002	0.6092
3	varied	-0.0122	0.1000	0.0852	0.7916
3	60	-0.0102	0.1007	0.0661	0.7768
3	100	-0.0054	0.0767	0.0662	0.8175
20	20	0.0065	0.0663	0.0220	0.7840
20	varied	-0.0008	0.0385	0.0522	0.8554
20	60	-0.0023	0.0365	0.0641	0.8707
20	100	-0.0027	0.0298	0.0481	0.8826

Table 3: Monte Carlo Results for Cost Instruments (10 Products per Firm,  $\beta = (3, 6)$ )

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